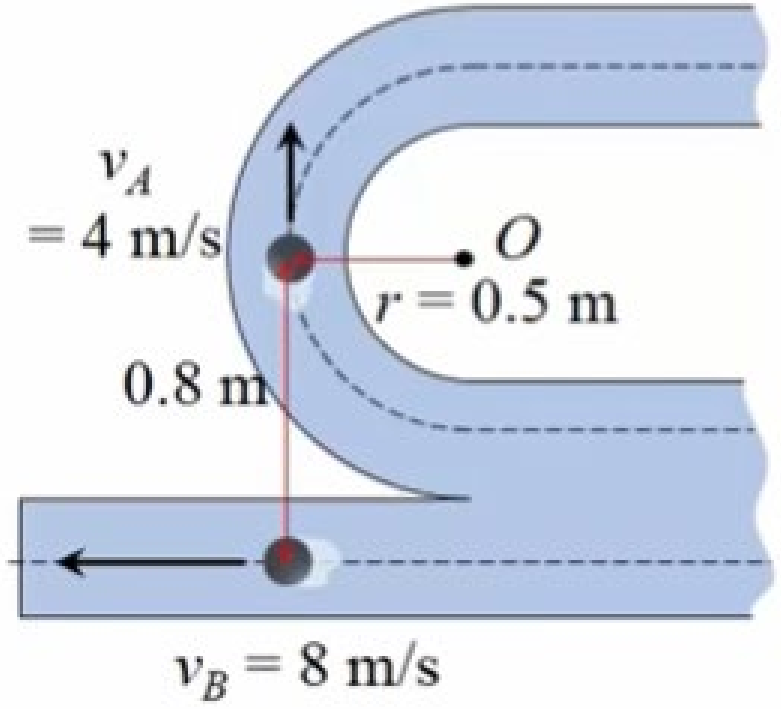
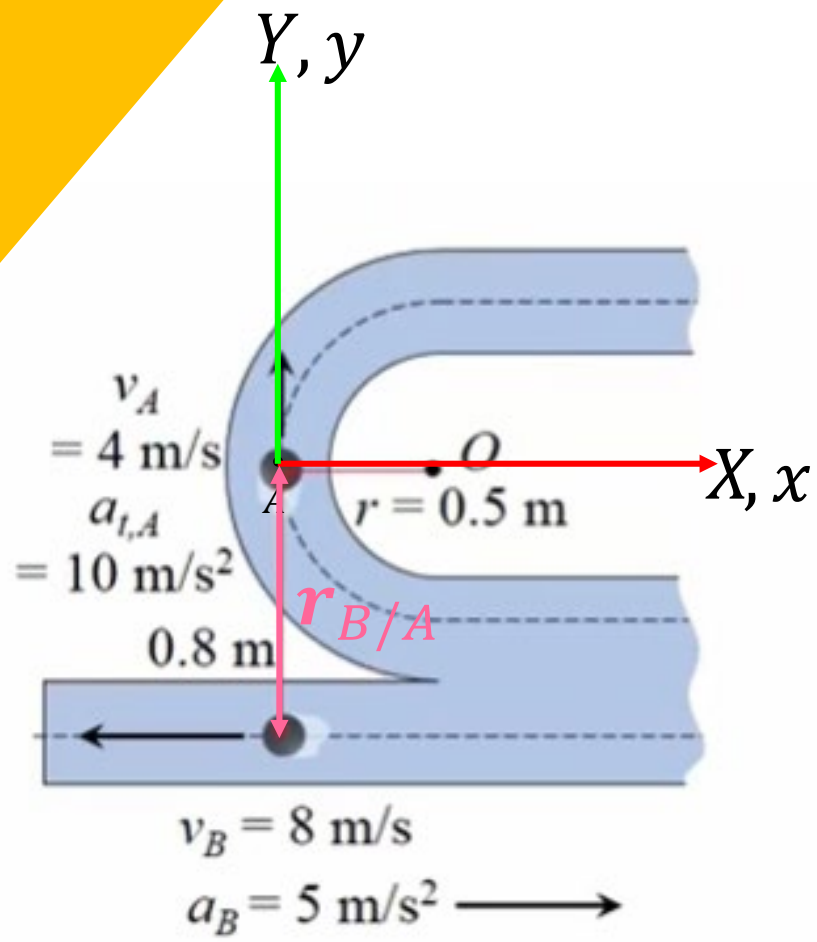
FRA131 Basic Mechanics
Rigid Body Kinematics
Relative-Motion Analysis using Rotating Axes: Examples

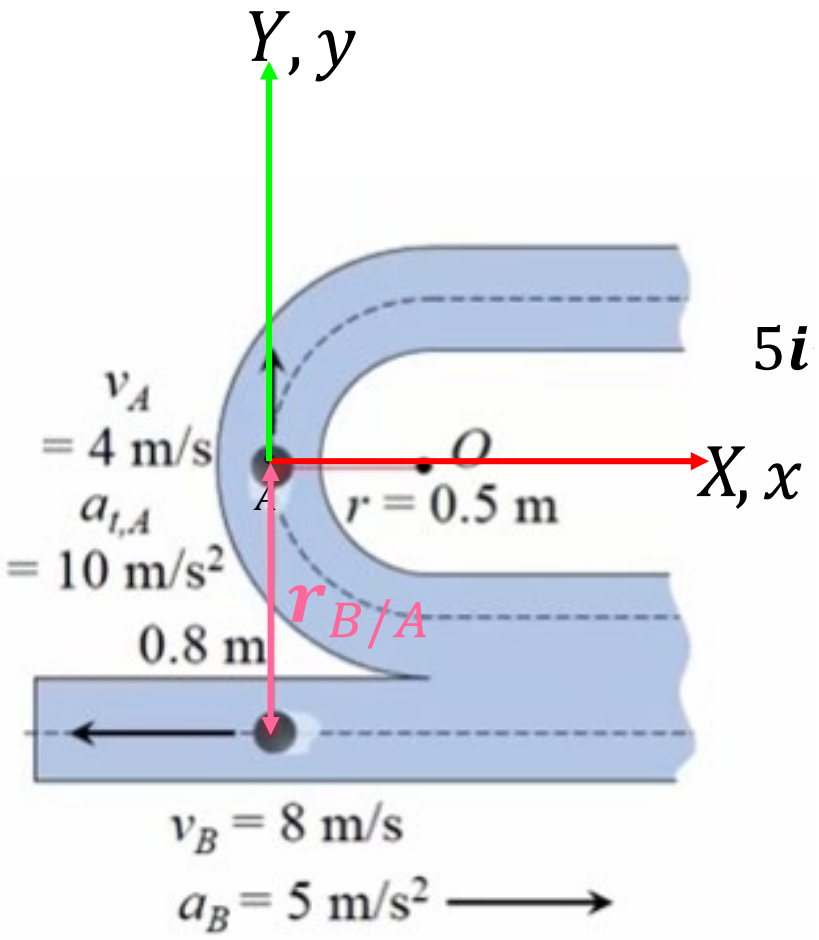
Boontariga KASEMSONTITUM, Ph.D.
(ดร.บุญฑริกา เกษมสันติธรรม)
Institute of Field roBOtics (FIBO)
King Mongkut's University of Technology Thonburi

Example 2 :

At the instant shown, ***took took*** *is travelling along the circular path and its speed is* ***increasing*** *at* $10\frac{m}{s^2}$; ***motorbike*** *is travelling along the straight path and its speed is* ***decreasing*** *at* $5\frac{m}{s^2}$. *Determine the relative velocity and accerelation of* B *with respect to* A.

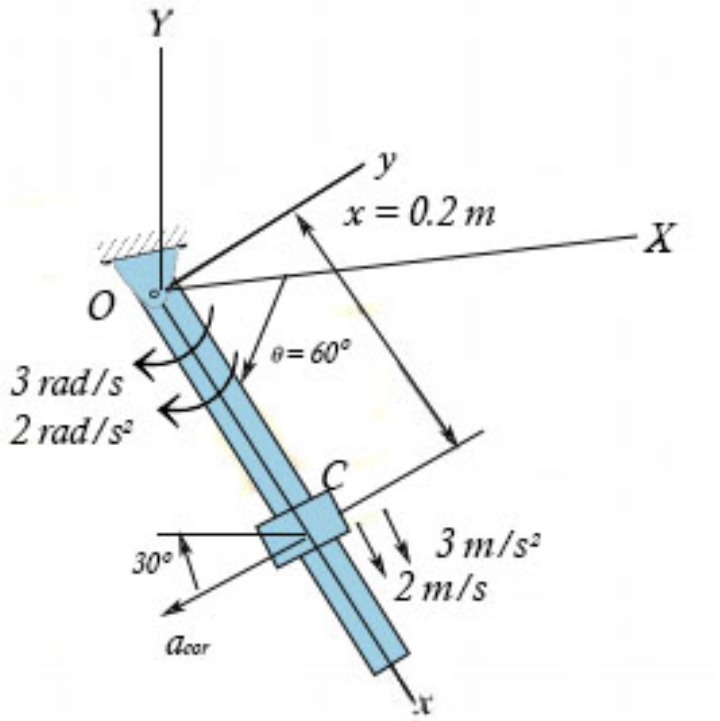


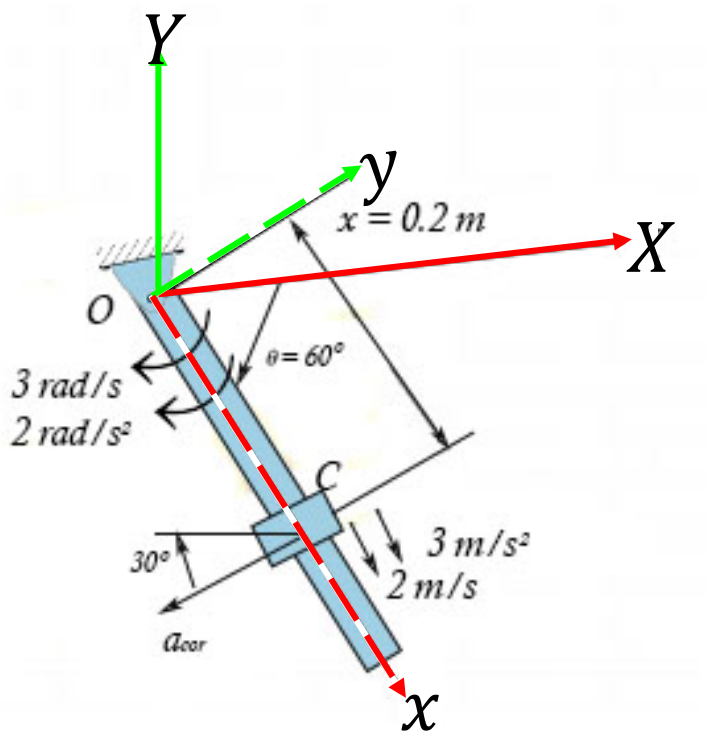

Y, y
X, x
O
A
r = 0.5 m
v_A
= 4 m/s
$a_{t,A}$
= 10 m/s²
0.8 m
$r_{B/A}$
v_B = 8 m/s
a_B = 5 m/s²

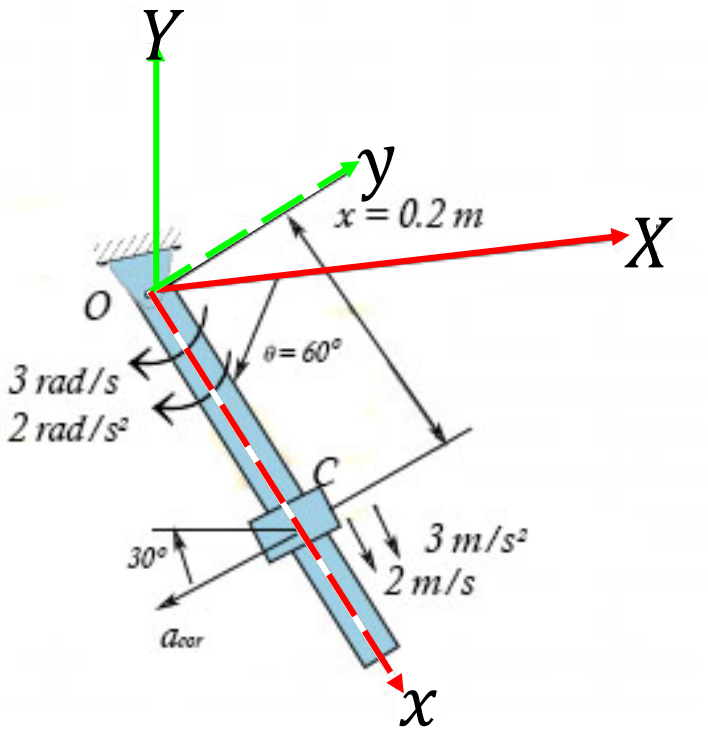

Y, y
5i
v_A
= 4 m/s
O
X, x
A
r = 0.5 m
$a_{t,A}$
= 10 m/s²
$r_{B/A}$
0.8 m
v_B = 8 m/s
a_B = 5 m/s²

Example 3 :

At the instant $\theta = 60^o$, *the rod in shown has an angular velocity of* $3\ rad/s$ *and an angular acceleration of* $2\frac{rad}{s^2}$. *At this same instant, collar C travels outward along the rod such that when* $x = 0.2\ m$ *the velocity is* $2\frac{m}{s}$ *and the acceleration is* $3\frac{m}{s^2}$, *both measured relative to the rod. Determine the Coriolis acceleration and the velocity and acceleration of the collar at this instant*

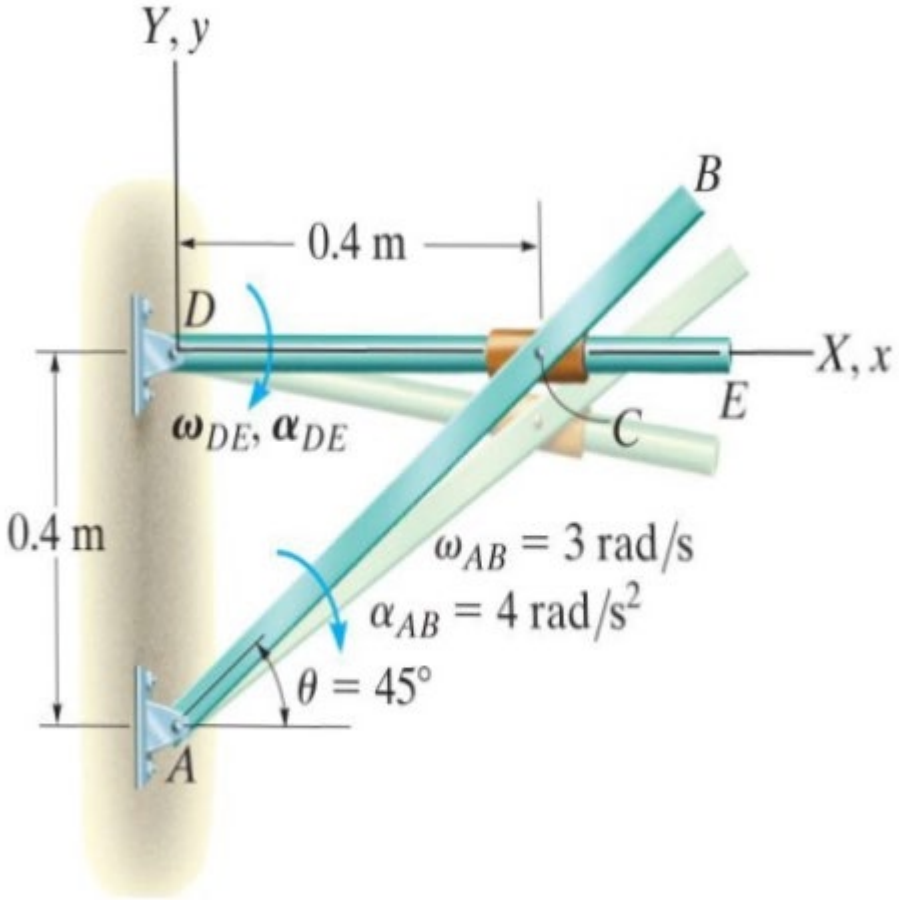


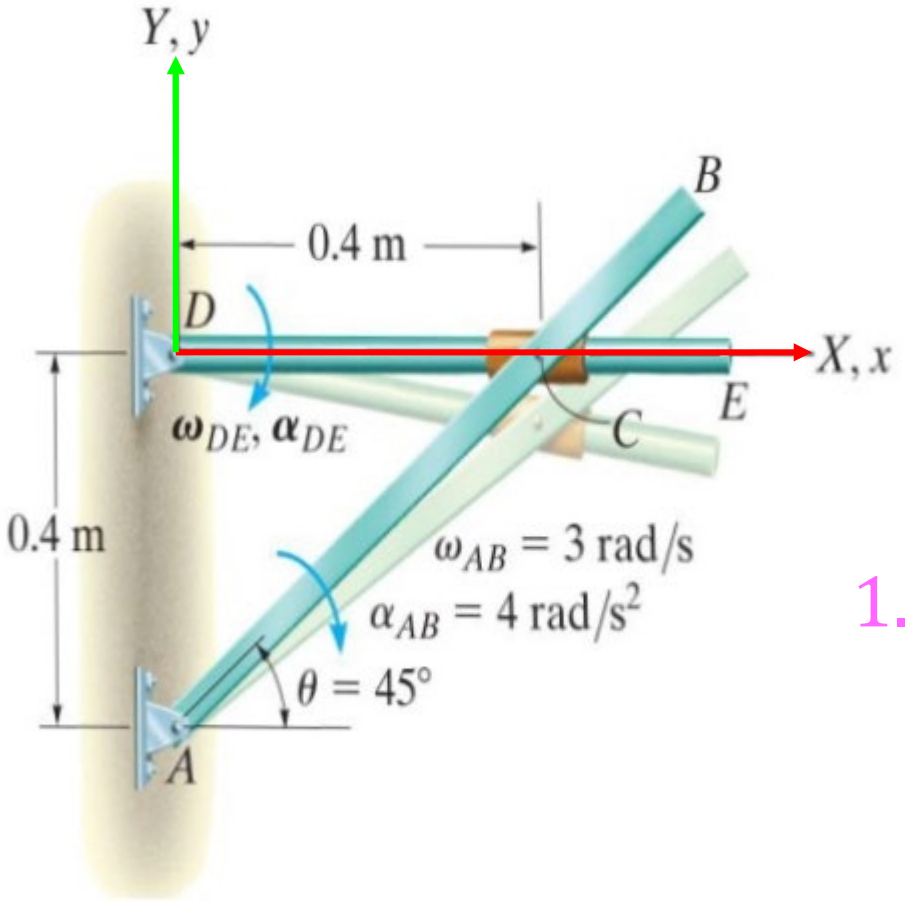

Y
y
x = 0.2 m
X
O
θ = 60°
3 rad/s
2 rad/s²
C
3 m/s²
2 m/s
30°
a_cor
x


Y
y
x = 0.2 m
X
O
θ = 60°
3 rad/s
2 rad/s²
C
3 m/s²
2 m/s
30°
a_cor
x

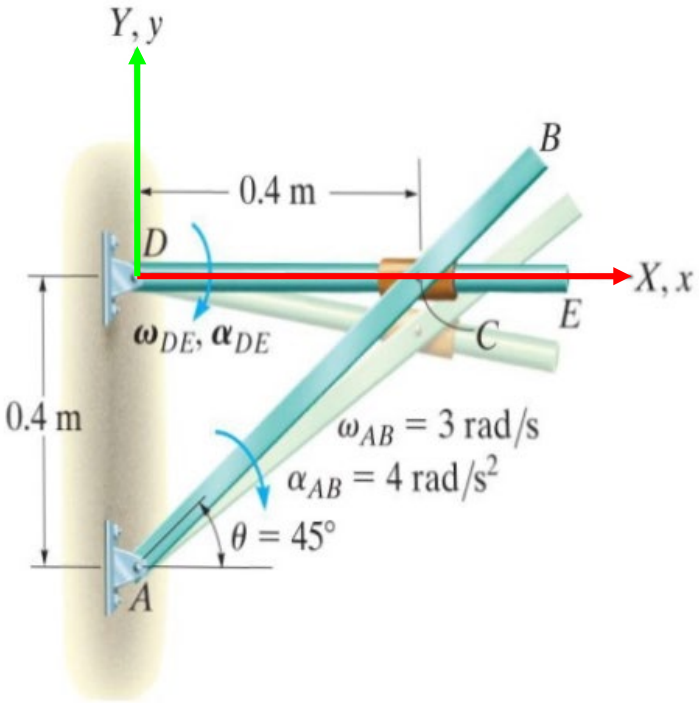
Example 4 :

Rod AB in shown, rotate clockwise such that it has an angular velocity $\omega_{AB} = 3\ rad/s$ *and angular acceleration* $\alpha_{AB} = 4\dfrac{rad}{s^2}$ *when* $\theta = 45^o$. *Determine the angular motion of rod DE at this instant. The collar at C is pin connected to AB and slides over rod DE.*



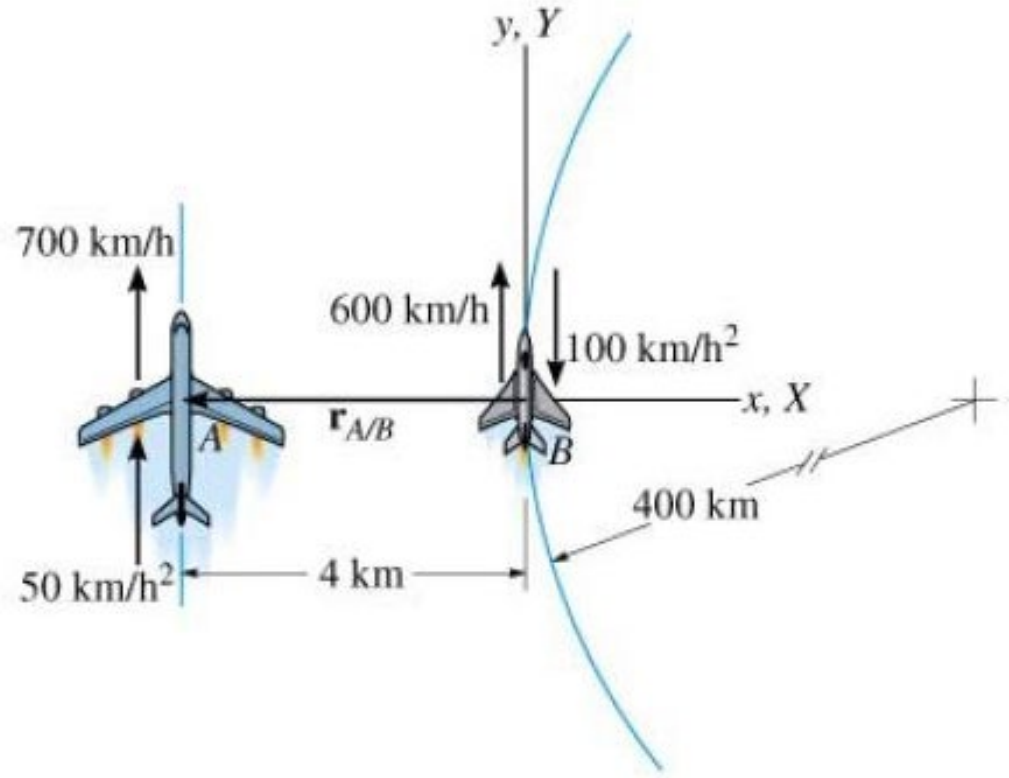

Y, y
B
0.4 m
D
X, x
E
C
ω_{DE}, α_{DE}
0.4 m
$\omega_{AB} = 3$ rad/s
$\alpha_{AB} = 4$ rad/s^2
$\theta = 45°$
A

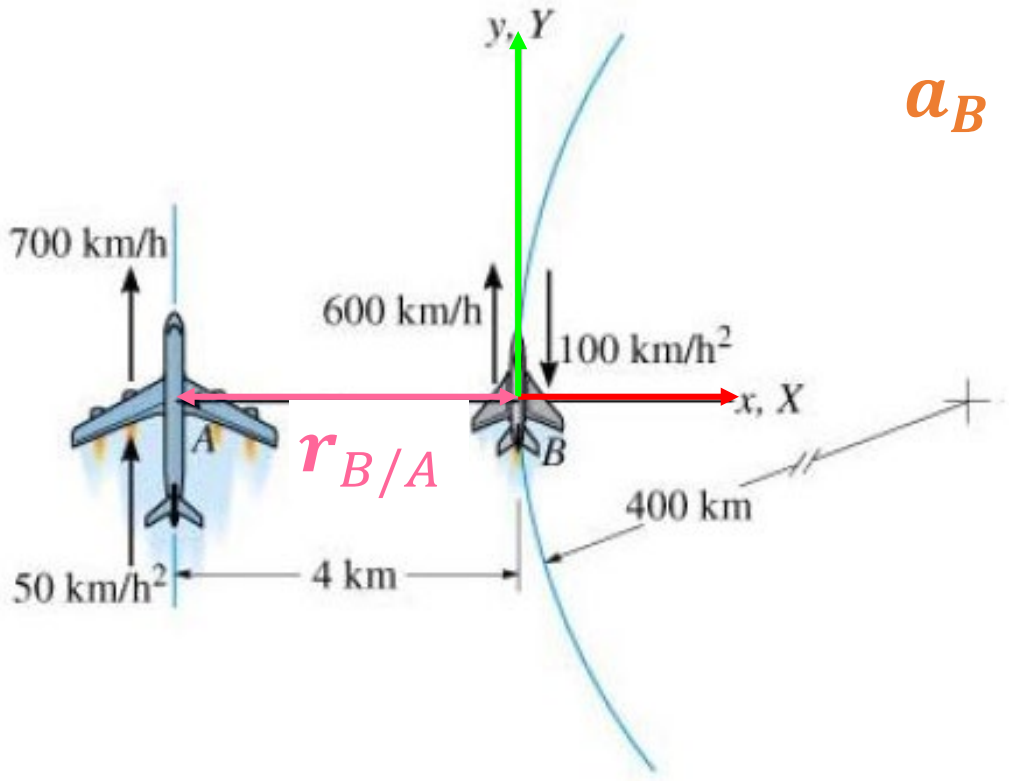
1.


Y, y
B
0.4 m
D
X, x
E
C
ω_{DE}, α_{DE}
0.4 m
$\omega_{AB} = 3$ rad/s
$\alpha_{AB} = 4$ rad/s^2
$\theta = 45°$
A

Example 5 :

Plane A and B fly at the same elevation and have the motions shown in shown. Determine the velocity and acceleration of A as measured by the pilot of B.





$$\boldsymbol{v_B} = \{600\boldsymbol{j}\}\ km/h$$

$$(\boldsymbol{a_B})_{\boldsymbol{n}} = \frac{\boldsymbol{v_B^2}}{\rho} = \frac{(600)^2}{400} = 900\ km/h^2$$

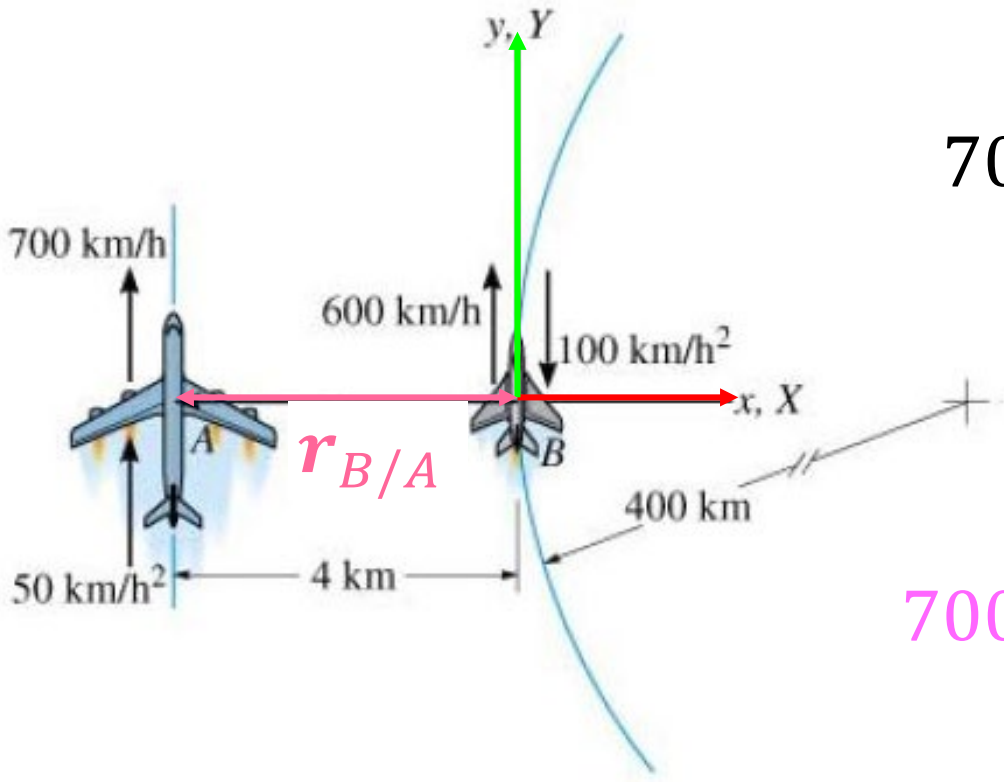
$$\boldsymbol{a_B} = (\boldsymbol{a_B})_{\boldsymbol{n}} + (\boldsymbol{a_B})_{\boldsymbol{t}} = \{900\boldsymbol{i} - 100\boldsymbol{j}\}\ km/h^2$$

$$\boldsymbol{\Omega} = \frac{\boldsymbol{v_B}}{\rho} = \frac{600}{400} = 1.5\ rad/h\ \circlearrowright$$

$$\boldsymbol{\Omega} = \{-1.5\mathbf{k}\}\ \text{rad/h}$$

$$\dot{\boldsymbol{\Omega}} = \frac{(\boldsymbol{a_B})_{\boldsymbol{t}}}{\rho} = \frac{100}{400} = 0.25\ rad/h^2\ \circlearrowleft$$

$$\dot{\boldsymbol{\Omega}} = \{0.25\ \boldsymbol{k}\}\ rad/h^2$$



$$v_A = v_B + \Omega \times r_{A/B} + (v_{A/B})_{xyz}$$

$700j$ $600j$ $-1.5k$ $-4i$

$$700j = 600j + (-1.5\mathbf{k}) \times (-4i) + (v_{A/B})_{xyz}$$

$$(v_{A/B})_{xyz} = \{94j\}\ (m/s)$$

$$a_A = a_B + \dot{\Omega} \times r_{A/B} - \Omega^2 r_{A/B} + 2\Omega \times (v_{A/B})_{xyz} + (a_{A/B})_{xyz}$$

a_A: $50j$

a_B: $900i - 100j$

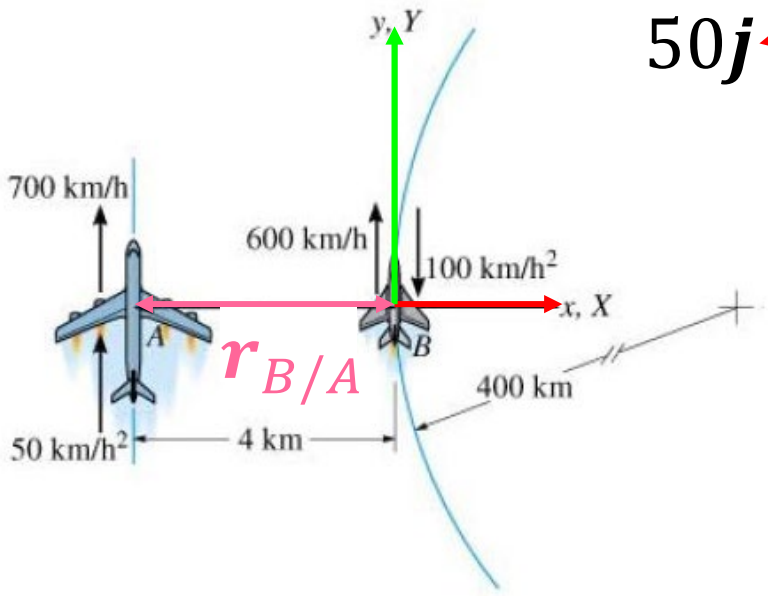
$\dot{\Omega}$: $0.25k$

$r_{A/B}$: $-4i$

Ω^2: -1.5

Ω: $-1.5k$

$(v_{A/B})_{xyz}$: $94j$



$$50j = (900i - 100j) + (0.25k) \times (-4i) - 1.5^2(-4i) + 2 \cdot (-1.5\mathbf{k}) \times 94j + (a_{A/B})_{xyz}$$

$$(a_{A/B})_{xyz} = \{-1191i + 151j\}\ km/h^2$$

Question 1 :

At the instant shown, rod BC has angular velocity and angular acceleration as shown. Link AC is pin – connected to collar C which slides freely on rod BC. *Determine the angular velocity and angular acc*eleration of link AC at this instant

Hint : the direcions of the absolute velocity and the relative velocity (wrt rod BC) of collar C are both known.